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Vacuum polarization by a global monopole with finite core

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Abstract: We investigate the effects of a $(D+1)$-dimensional global monopole core on the behavior of a quantum massive scalar field with general curvature coupling parameter. In the general case of the spherically symmetric static core, formulae are derived for the Wightman function, for the vacuum expectation values of the field square and the energy-momentum tensor in the exterior region. These expectation values are presented as the sum of point-like global monopole part and the core induced one. The asymptotic behavior of the core induced vacuum densities is investigated at large distances from the core, near the core and for small values of the solid angle corresponding to strong gravitational fields. In particular, in the latter case we show that the behavior of the vacuum densities is drastically different for minimally and non-minimally coupled fields. As an application of general results the flower-pot model for the monopole's core is considered and the expectation values inside the core are evaluated.

Keywords: Solitons Monopoles and Instantons, Field Theories in Higher Dimensions.

Contents

1. Introduction

It is well known that different types of topological objects may have been formed in the early universe after Planck time by the vacuum phase transition [1, 2]. Depending on the topology of the vacuum manifold these are domain walls, strings, monopoles and textures. Among them, cosmic strings and monopoles seem to be the best candidate to be observed. A global monopole is a spherical heavy object formed in the phase transition of a system composed by a self-coupling Goldstone field, whose original global symmetry is spontaneously broken. The matter fields play the role of an order parameter which outside the monopole's core acquires a non-vanishing value. The global monopole was first introduced by Sokolov and Starobinsky [3]. A few years later, the gravitational effects of of the global monopole were considered in ref. [4], where a solution is presented which describes a global monopole at large radial distances. The gravitational effects produced by this object may be approximated by a solid angle deficit in the (3+1)-dimensional spacetime.

The nontrivial properties of the vacuum are among the most important predictions of quantum field theory. These properties are manifested in the response of the vacuum to the external electromagnetic and gravitational fields. In particular, the explicit calculations of the vacuum polarization caused by particular external fields have played an important role in the development of quantum field theory. The quantum effects due to the point-like global monopole spacetime on the matter fields have been considered for massless scalar [5] and fermionic [6] fields, respectively. In order to develop this analysis, the scalar respectively spinor Green functions in this background have been obtained. The influence of the non-zero temperature on these polarization effects has been considered in [7] for scalar and fermionic fields. Moreover, the calculation of quantum effects on massless scalar field in a higher dimensional global monopole spacetime has also been developed in [8]. The combined vacuum polarization effects by the non-trivial geometry of a global monopole and boundary conditions imposed on the matter fields are investigated as well. In this direction, the total Casimir energy associated with massive scalar field inside a spherical region in the global monopole background have been analyzed in refs. [9, 10] by using the zeta function regularization procedure. Scalar Casimir densities induced by spherical boundaries have been calculated in [11, 12] to higher dimensional global monopole spacetime by making use of the generalized Abel-Plana summation formula [13, 14]. More recently, using also this formalism, a similar analysis for spinor fields with MIT bag boundary conditions has been developed in [15, 16].

Many of treatments of quantum fields around a global monopole deal mainly with the case of the idealized point-like monopole geometry. However, the realistic global monopole has a characteristic core radius determined by the symmetry braking scale at which the monopole is formed. A simplified model for the monopole core where the region inside the core is described by the de Sitter geometry is presented in [17]. The vacuum polarization effects due to a massless scalar field in the region outside the core of this model are investigated in ref. [18]. In particular, it has been shown that long-range effects can take place due to the non-trivial core structure. In the present paper we will analyze the effects of global monopole core on properties of the quantum vacuum for the general spherically symmetric static model with a core of finite radius. The most important quantities characterizing these properties are the vacuum expectation values of the field square and the energy-momentum tensor. Though the corresponding operators are local, due to the global nature of the vacuum, the vacuum expectation values describe the global properties of the bulk and carry an important information about the structure of the defect core. In addition to describing the physical structure of the quantum field at a given point, the energy-momentum tensor acts as the source of gravity in the Einstein equations. It therefore plays an important role in modelling a self-consistent dynamics involving the gravitational field. As the first step for the investigation of vacuum densities we evaluate the positive frequency Wightman function for a massive scalar field with general curvature coupling parameter. This function gives comprehensive insight into vacuum fluctuations and determines the response of a particle detector of the Unruh-DeWitt type moving in the global monopole bulk. The problem under consideration is also of separate interest as an example with gravitational and boundary-induced polarizations of the vacuum, where all calculations can be performed in a closed form. The corresponding results specify the conditions under which we can ignore the details of the interior structure and approximate the effect of the global monopole by the idealized model.

The paper is organized as follows. In section 2 we consider the Wightman function in the exterior of the global monopole for the general structure of the core assuming that the components of the metric tensor and their derivatives are continuous at the transition surface between the core and the exterior. By using this function, in section 3 we investigate the vacuum expectation values of the field square and the energy-momentum tensor. The

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section 4 is devoted to the generalization of the corresponding results when an additional surface shell is present on the bounding surface between the core and the exterior. As an illustration of the general results, in section 5 we consider the flower-pot model with the Minkowskian geometry inside the core. For this model the vacuum expectation values inside the core are investigated as well. In section 6 we present our concluding remarks. In appendix we show that the formulae obtained in the paper for the core induced parts are also valid in the case when bound states are present.

2. Wightman function

We consider a model of $(D+1)$-dimensional global monopole with a core of radius a in which the spacetime is described by two distinct metric tensors in the regions outside and inside the core. In the hyperspherical polar coordinates $(r, \vartheta, \phi) \equiv (r, \theta_1, \theta_2, \ldots \theta_n, \phi)$, $n = D - 2$, the corresponding line element in the exterior region $r > a$ has the form

$$ds^2 = dt^2 - dr^2 - \sigma^2 r^2 d\Omega_D^2, \tag{2.1}$$

where $d\Omega_D^2$ is the line element on the surface of the unit sphere in D-dimensional Euclidean space, the parameter σ is smaller than unity and is related to the symmetry breaking energy scale in the theory. The solid angle corresponding to eq. (2.1) is $\sigma^2 S_D$ with $S_D = 2\pi^{D/2}/\Gamma(D/2)$ being the total area of the surface of the unit sphere in D-dimensional Euclidean space. This leads to the solid angle deficit $(1 - \sigma^2)S_D$ in the spacetime given by line element (2.1). It is of interest to note that the effective metric produced in superfluid $^3\mathrm{He} - \mathrm{A}$ by a monopole is described by the three dimensional version of the line element (2.1) with the negative angle deficit, $\sigma > 1$, which corresponds to the negative mass of the topological object [19]. The quasiparticles in this model are chiral and massless fermions. We will assume that inside the core (region $r < a$) the spacetime geometry is regular and is described by the general static spherically symmetric line element

$$ds^2 = e^{2u(r)} dt^2 - e^{2v(r)} dr^2 - e^{2w(r)} d\Omega_D^2, \tag{2.2}$$

where the functions $u(r)$, $v(r)$, $w(r)$ are continuous at the core boundary:

$$u(a) = v(a) = 0, \; w(a) = \ln(\sigma a). \tag{2.3}$$

Here we assume that there is no surface energy-momentum tensor located at $r = a$ and, hence, the derivatives of these functions are continuous as well. The generalization to the case with an infinitely thin spherical shell at the boundary of two metrics will be discussed in section 4. Note that by introducing the new radial coordinate $\tilde{r} = e^{w(r)}$ with the core center at $\tilde{r} = 0$, the angular part of the line element (2.2) is written in the standard Minkowskian form. With this coordinate, in general, we will obtain non-standard angular part in the exterior line element (2.1). For the metric corresponding to line element (2.2) the nonzero components of the Ricci tensor are given by expressions (no summation over i, we adopt the convention of Birrell and Davies [20] for the curvature tensor)

$$R_0^0 = -e^{-2v} \left[u'' + u'^2 - u'v' + (n+1)u'w' \right],$$

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$$\begin{aligned} R_1^1 &= -e^{-2v}\left[u'' + u'^2 - u'v' + (n+1)\left(w'' + w'^2 - w'v'\right)\right], \\ R_i^i &= -e^{-2v}\left(w'' + w'^2 + w'u' - w'v' + nw'^2\right) + ne^{-2w}, \end{aligned} \tag{2.4}$$

where the prime means the derivative with respect to the radial coordinate r and the indices $i = 2, 3, \ldots, D$ correspond to the coordinates $\theta_1, \theta_2, \ldots, \phi$ respectively. The corresponding Ricci scalar has the form

$$\begin{aligned} R = &-2e^{-2v}\left[u'' + u'^2 - u'v' + n(n+1)w'^2/2 \right. \\ &\left. +(n+1)\left(w'' + w'^2 + w'u' - w'v'\right)\right] + n(n+1)e^{-2w}. \end{aligned} \tag{2.5}$$

Note that from the regularity of the interior geometry at the core center one has the conditions $u(r), v(r) \to 0$, and $w(r) \sim \ln \tilde{r}$ for $\tilde{r} \to 0$. In the region outside the core, $r > a$, for the nonzero components we have the standard expressions (no summation over i):

$$R_i^i = n\frac{1-\sigma^2}{\sigma^2 r^2}, \quad R = n(n+1)\frac{1-\sigma^2}{\sigma^2 r^2}, \tag{2.6}$$

where $i = 2, 3, \ldots, D$. For $n = 0$ the spacetime outside the core is flat and coincides with $D = 2$ cosmic string geometry. The influence of the non-trivial core structure for the cosmic string on a quantum scalar field has been considered in refs. [21 − 23]. In the discussion below we will assume that $n > 0$.

In this paper we are interested in the vacuum polarization effects for a scalar field with general curvature coupling parameter ξ propagating in the bulk described above. The corresponding field equation has the form

$$\left(\nabla_i \nabla^i + m^2 + \xi R\right)\varphi = 0, \tag{2.7}$$

where ∇_i is the covariant derivative operator associated with line element (2.1) outside the core and with line element (2.2) inside the core. The values of the curvature coupling parameter $\xi = 0$, and $\xi = \xi_D$ with $\xi_D \equiv (D-1)/4D$ correspond to the most important special cases of minimally and conformally coupled scalar fields, respectively. As a first stage for the evaluation of the vacuum expectation values (VEVs) for the field square and the energy-momentum tensor we consider the positive frequency Wightman function $\langle 0|\varphi(x)\varphi(x')|0\rangle$, where $|0\rangle$ is the amplitude for the corresponding vacuum state. This function also determines the response of the Unruh-DeWitt type particle detector at a given state of motion (see, for instance, [20]). By expanding the field operator over eigenfunctions and using the commutation relations one can see that

$$\langle 0|\varphi(x)\varphi(x')|0\rangle = \sum_\alpha \varphi_\alpha(x)\varphi_\alpha^*(x'), \tag{2.8}$$

with $\{\varphi_\alpha(x), \varphi_\alpha^*(x')\}$ being a complete orthonormalized set of positive and negative frequency solutions to the field equation. The collective index α can contain both discrete and continuous components. In eq. (2.8) it is assumed summation over discrete indices and integration over continuous indices.

Due to the symmetry of the problem under consideration the eigenfunctions can be presented in the form

$$\varphi_\alpha(x) = f_l(r)Y(m_k;\vartheta,\phi)e^{-i\omega t},\ l = 0,1,2,\dots, \tag{2.9}$$

where $m_k = (m_0 \equiv l, m_1, \dots, m_n)$, and $m_1, m_2, \dots, m_n$ are integers such that

$$0 \le m_{n-1} \le m_{n-2} \le \cdots \le m_1 \le l, \quad -m_{n-1} \le m_n \le m_{n-1}, \tag{2.10}$$

and $Y(m_k;\vartheta,\phi)$ is the hyperspherical harmonic of degree l [24]. The equation for the radial function is obtained from the field equation (2.7) and has the form

$$f_l''(r) + \left[u' - v' + (D-1)w'\right] f_l'(r) + e^{2v}\left[e^{-2u}\omega^2 - m^2 - \xi R - l(l+n)e^{-2w}\right] f_l(r) = 0. \tag{2.11}$$

In the region $r > a$ described by the line element (2.1), the linearly independent solutions to this equation are $r^{-n/2}J_{\nu_l}(\lambda r)$ and $r^{-n/2}Y_{\nu_l}(\lambda r)$ with $\lambda = \sqrt{\omega^2 - m^2}$, where $J_{\nu_l}(x)$ and $Y_{\nu_l}(x)$ are the Bessel and Neumann functions with the order

$$\nu_l = \frac{1}{\sigma}\left[\left(l + \frac{n}{2}\right)^2 + (1-\sigma^2)n(n+1)\left(\xi - \xi_{D-1}\right)\right]^{\frac{1}{2}}. \tag{2.12}$$

In the following consideration we will assume that ν_l^2 is non-negative. This corresponds to the restriction on the values of the curvature coupling parameter for $n > 0$, given by the condition

$$(1-\sigma^2)\xi \geqslant -\frac{n\sigma^2}{4(n+1)}. \tag{2.13}$$

This condition is satisfied by the minimally coupled field for all values σ and by the conformally coupled field for $\sigma \leqslant D-1$. The solution of the radial equation (2.11) in the region $r < a$ regular at the origin we will denote by $R_l(r,\lambda)$. From eq. (2.11) it follows that near the core center this solution behaves as $\tilde{r}^l$. Note that the parameter λ enters in the radial equation in the form λ^2. As a result the regular solution can be chosen in such a way that $R_l(r,-\lambda) = \text{const}\cdot R_l(r,\lambda)$. Now for the radial part of the eigenfunctions one has

$$f_l(r) = \begin{cases} R_l(r,\lambda) & \text{for } r < a \\ r^{-n/2}\left[A_l J_{\nu_l}(\lambda r) + B_l Y_{\nu_l}(\lambda r)\right] & \text{for } r > a \end{cases}, \tag{2.14}$$

where the coefficients A_l and B_l are determined by the conditions of continuity of the radial function and its derivative at $r = a$. From these conditions we find the following expressions for these coefficients

$$A_l = \frac{\pi}{2}a^{n/2}R_l(a,\lambda)\bar{Y}_{\nu_l}(\lambda a),\ B_l = -\frac{\pi}{2}a^{n/2}R_l(a,\lambda)\bar{J}_{\nu_l}(\lambda a). \tag{2.15}$$

Here and in what follows for a cylinder function $F(z)$ we use the notation

$$\bar{F}(z) \equiv zF'(z) - \left[\frac{n}{2} + a\frac{R_l'(a,z/a)}{R_l(a,z/a)}\right]F(z), \tag{2.16}$$

where $R_l'(a,\lambda) = \partial R_l(r,\lambda)/\partial r|_{r=a}$. Note that due to our choice of the function $R_l(r,\lambda)$, the logarithmic derivative in formula (2.16) is an even function on z. Hence, in the region $r > a$ the radial part of the eigenfunctions has the form

$$f_l(r) = \frac{\pi a^{n/2}}{2r^{n/2}} R_l(a,\lambda) g_{\nu_l}(\lambda a, \lambda r), \tag{2.17}$$

where the notation

$$g_{\nu_l}(\lambda a, \lambda r) = J_{\nu_l}(\lambda r)\bar{Y}_{\nu_l}(\lambda a) - \bar{J}_{\nu_l}(\lambda a) Y_{\nu_l}(\lambda r). \tag{2.18}$$

is introduced.

For the eigenfunctions we have the following orthonormalization condition

$$\int dV \sqrt{-g} g^{00} \varphi_\alpha(x) \varphi_{\alpha'}^*(x) = \frac{\delta_{\alpha\alpha'}}{2\omega}, \tag{2.19}$$

where $\delta_{\alpha\alpha'}$ is understood as the Kronecker symbol for discrete indices and as the Dirac delta function for continuous ones. Substituting eigenfunctions (2.9), and using the relation

$$\int d\Omega \, |Y(m_k;\vartheta,\phi)|^2 = N(m_k) \tag{2.20}$$

(the explicit form for $N(m_k)$ is given in [24] and will not be necessary for the following consideration in this paper), the normalization condition is written in terms of the radial eigenfunctions

$$\int_{r_0}^{\infty} dr \sqrt{-g_r} g^{00} f_l(r,\lambda) f_l(r,\lambda') = \frac{\delta(\lambda - \lambda')}{2\omega N(m_k)}, \tag{2.21}$$

where r_0 is the value of the radial coordinate r corresponding to the origin and g_r is the radial part of the determinant g. Note that in general $r_0 \neq 0$ (see, for instance, the special case of the flower-pot model in section 5). As the integral on the left is divergent for $\lambda' = \lambda$, the main contribution in the coincidence limit comes from large values r. By using the expression (2.17) for the radial part in the region $r > a$ and replacing the Bessel and Neumann functions by the leading terms of their asymptotic expansions for large values of the argument, it can be seen that from (2.21) the following result is obtained:

$$a^n R_l^2(a,\lambda) = \frac{2\sigma^{1-D}\lambda}{\pi^2 \omega N(m_k) \left[\bar{J}_{\nu_l}^2(\lambda a) + \bar{Y}_{\nu_l}^2(\lambda a)\right]}. \tag{2.22}$$

Having the normalized eigenfunctions, now we turn to the evaluation of the Wightman function by using the mode sum formula (2.8). Substituting eigenfunctions (2.17) and using the addition formula for the hyperspherical harmonics [24]

$$\sum_{m_k} \frac{Y(m_k;\vartheta,\phi)}{N(m_k)} Y^*(m_k;\vartheta',\phi') = \frac{2l+n}{nS_D} C_l^{n/2}(\cos\theta), \tag{2.23}$$

for the Wightman function in the region outside the monopole's core one obtains

$$\langle 0|\varphi(x)\varphi(x')|0\rangle = \frac{\sigma^{1-D}}{2nS_D} \sum_{l=0}^{\infty} \frac{2l+n}{(rr')^{n/2}} C_l^{n/2}(\cos\theta)$$

$$\times \int_0^\infty \frac{\lambda d\lambda}{\sqrt{\lambda^2+m^2}} \frac{g_{\nu_l}(\lambda a, \lambda r) g_{\nu_l}(\lambda a, \lambda r')}{\bar{J}_{\nu_l}^2(\lambda a) + \bar{Y}_{\nu_l}^2(\lambda a)} e^{i\sqrt{\lambda^2+m^2}(t'-t)}. \tag{2.24}$$

In formula (2.23), $S_D = 2\pi^{D/2}/\Gamma(D/2)$ is the total area of the surface of the unit sphere in D-dimensional space, $C_p^q(x)$ is the Gegenbauer or ultraspherical polynomial of degree p and order q, and θ is the angle between directions (ϑ, ϕ) and (ϑ', ϕ'). Let us denote by $\langle 0_{\rm m}|\varphi(x)\varphi(x')|0_{\rm m}\rangle$ the positive frequency Wightman function for the geometry of the idealized point-like global monopole described by the line element (2.1) for all values of the radial coordinate. This function can be presented in the form [11]

$$\langle 0_{\rm m}|\varphi(x)\varphi(x')|0_{\rm m}\rangle = \frac{\sigma^{1-D}}{2nS_D} \sum_{l=0}^\infty \frac{2l+n}{(rr')^{n/2}} C_l^{n/2}(\cos\theta) \times \int_0^\infty d\lambda \, \frac{\lambda e^{i\sqrt{\lambda^2+m^2}(t'-t)}}{\sqrt{\lambda^2+m^2}} J_{\nu_l}(\lambda r) J_{\nu_l}(\lambda r'). \tag{2.25}$$

In order to investigate the part induced by the non-trivial core structure, we consider the difference

$$\langle \varphi(x)\varphi(x')\rangle_c = \langle 0|\varphi(x)\varphi(x')|0\rangle - \langle 0_{\rm m}|\varphi(x)\varphi(x')|0_{\rm m}\rangle. \tag{2.26}$$

Using formulae (2.24), (2.25) and the relation

$$\frac{g_\nu(\lambda a, \lambda r) g_\nu(\lambda a, \lambda r')}{\bar{J}_\nu^2(\lambda a) + \bar{Y}_\nu^2(\lambda a)} - J_\nu(\lambda r) J_\nu(\lambda r') = -\frac{1}{2} \sum_{s=1}^2 \frac{\bar{J}_\nu(\lambda a)}{\bar{H}_\nu^{(s)}(\lambda a)} H_\nu^{(s)}(\lambda r) H_\nu^{(s)}(\lambda r'), \tag{2.27}$$

with $H_\nu^{(s)}(x)$, $s = 1, 2$ being the Hankel functions, the core induced part in the Wightman function is presented in the form

$$\langle \varphi(x)\varphi(x')\rangle_c = -\frac{\sigma^{1-D}}{4nS_D} \sum_{l=0}^\infty \frac{2l+n}{(rr')^{n/2}} C_l^{n/2}(\cos\theta) \sum_{s=1}^2 \int_0^\infty d\lambda \, \lambda \times \frac{e^{i\sqrt{\lambda^2+m^2}(t'-t)}}{\sqrt{\lambda^2+m^2}} \frac{\bar{J}_{\nu_l}(\lambda a)}{\bar{H}_{\nu_l}^{(s)}(\lambda a)} H_{\nu_l}^{(s)}(\lambda r) H_{\nu_l}^{(s)}(\lambda r'). \tag{2.28}$$

Now we rotate the integration contour in the complex plane λ by the angle $\pi/2$ for $s = 1$ and by the angle $-\pi/2$ for $s = 2$. By using the property that the logarithmic derivative of the function $R_l(r, \lambda)$ in formula (2.16) is an even function on z, we can see that the integrals over the segments $(0, im)$ and $(0, -im)$ of the imaginary axis cancel out. As a result, after introducing the modified Bessel functions, the core induced part can be presented in the form

$$\langle \varphi(x)\varphi(x')\rangle_c = -\frac{\sigma^{1-D}}{\pi nS_D} \sum_{l=0}^\infty \frac{2l+n}{(rr')^{n/2}} C_l^{n/2}(\cos\theta) \int_m^\infty dz \, z \frac{\tilde{I}_{\nu_l}(za)}{\tilde{K}_{\nu_l}(za)} \times \frac{K_{\nu_l}(zr) K_{\nu_l}(zr')}{\sqrt{z^2-m^2}} \cosh\left[\sqrt{z^2-m^2}(t'-t)\right]. \tag{2.29}$$

Here and below the tilted notation for the modified Bessel functions is defined as

$$\tilde{F}(z) \equiv zF'(z) - \mathcal{R}_l(a, z) F(z), \tag{2.30}$$

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with

$$\mathcal{R}_l(a,z) = \frac{n}{2} + a\frac{R_l'(a, ze^{\pi i/2}/a)}{R_l(a, ze^{\pi i/2}/a)}. \tag{2.31}$$

The VEVs in the bulk of the idealized point-like global monopole are well-investigated in literature (see, for instance, [5]–[12] and references therein) and in the discussion below we will be mainly concerned with the part induced by the non-trivial core structure. As we see from (2.29), all information about the inner structure of the global monopole is contained in the logarithmic derivative of the interior radial function in formula (2.31). In deriving formula (2.29) we have assumed that there are no bound states for which λ is purely imaginary. In appendix we show that this formula is also valid in the case when bound states are present.

3. Vacuum expectation values outside the monopole core

The VEV of the field square is obtained by computing the Wightman function in the coincidence limit $x' \to x$. In this limit expression (2.24) gives a divergent result and some renormalization procedure is needed. Outside the monopole core the local geometry is the same as that for a point-like global monopole. Hence, in the region $r > a$ the renormalization procedure for the local characteristics of the vacuum, such as the field square and the energy-momentum tensor, is the same as for the point-like global monopole geometry. This procedure is discussed in a number of papers (see [5]–[8]). For the renormalization we must subtract the corresponding DeWitt-Schwinger expansion involving the terms up to order D. For a massless field the renormalized value of the field square has the structure $\langle\varphi^2\rangle_{\mathrm{m,ren}} = [A + B\ln(\mu r)]/r^{D-1}$, where the coefficients A and B are functions on the parameters σ and ξ only and the arbitrary mass scale μ corresponds to the ambiguity in the renormalization procedure. For a spacetime of odd dimension $B = 0$ and this ambiguity is absent. In general, it is not possible to obtain closed expression for the coefficients A and B. For small values $1-\sigma^2$ approximate expressions are derived in ref. [8] for $D = 4$ and $D = 5$. In this paper our main interest are the parts in the VEVs induced by the non-trivial core structure and below we will concentrate on these quantities.

By using the formula for the Wightman function from the previous section, the VEV of the field square in the exterior region is presented in the form

$$\langle\varphi^2\rangle_{\mathrm{ren}} = \langle\varphi^2\rangle_{\mathrm{m,ren}} + \langle\varphi^2\rangle_c$$

Taking into account the relation

$$C_l^{n/2}(1) = \frac{\Gamma(l+n)}{\Gamma(n)l!}, \tag{3.1}$$

for the part induced by the core we find

$$\langle\varphi^2\rangle_c = -\frac{\sigma^{1-D}}{\pi r^n S_D}\sum_{l=0}^{\infty} D_l \int_m^{\infty} dz\, z\frac{\tilde{I}_{\nu_l}(za)}{\tilde{K}_{\nu_l}(za)}\frac{K_{\nu_l}^2(zr)}{\sqrt{z^2-m^2}}. \tag{3.2}$$

Here the factor

$$D_l = (2l + D - 2)\frac{\Gamma(l + D - 2)}{\Gamma(D - 1)\, l!} \tag{3.3}$$

is the degeneracy of each angular mode with given l. For a fixed l and large z the integrand contains the exponential factor $e^{2z(a-r)}$ and the integral converges when $r > a$. For large values l, introducing a new integration variable $y = z/\nu_l$ in the integral of eq. (3.2) and using the uniform asymptotic expansions for the modified Bessel functions [25], it can be seen that the both integral and sum are convergent for $r > a$ and diverge at $r = a$. For the points near the sphere the part (3.2) behaves as $1/(r-a)^{\beta_1}$, where β_1 is an integer which depends on the specific model of the core. For this parameter one has $\beta_1 \leqslant D - 1$. The exception is the case of the core model for which the leading term in the uniform asymptotic expansion of the function $\tilde{K}_{\nu_l}(za)$ for large values l vanishes. The latter takes place for the interior radial function with the asymptotic behavior $\mathcal{R}_l(a, lz/\sigma) \sim -(l/\sigma)\sqrt{1+z^2}$ for large l. For the case of a massless scalar the asymptotic behavior of the part (3.2) at large distances from the sphere can be obtained by introducing a new integration variable $y = zr$ and expanding the integrand in terms of a/r. The leading contribution for the summand with a given l has an order $(a/r)^{2\nu_l + D - 1}$ [assuming that $\nu_l \neq 0$ and $\mathcal{R}_l(a, 0) \neq \pm\nu_l$] and the main contribution comes from the $l = 0$ term. Now comparing this with the part $\langle\varphi^2\rangle_{\mathrm{m,ren}}$, we see that for $\nu_0 > 0$ the VEV of the field square at large distances from the core is dominated by the part corresponding to the geometry of the point-like global monopole. For the case $\nu_0 = 0$ the ratio $\langle\varphi^2\rangle_c/\langle\varphi^2\rangle_{\mathrm{m,ren}}$ decays logarithmically and long-range effects of the monopole core appear similar to those for the geometry of a cosmic string [21, 22] (see also the discussion in ref. [18] for the model with de Sitter spacetime inside the core). This case is realized by special values of the parameters satisfying the condition $(1/\sigma^2 - 1)\xi = -\xi_{D-1}$. For a massive field assuming that $mr \gg 1$, the main contribution into the integral over z in eq. (3.2) comes from the lower limit and to the leading order one has

$$\langle\varphi^2\rangle_c = -\frac{\sqrt{\pi}\sigma^{1-D} e^{-2mr}}{4r^{n+1} S_D \sqrt{mr}} \sum_{l=0}^{\infty} D_l \frac{\tilde{I}_{\nu_l}(ma)}{\tilde{K}_{\nu_l}(ma)}, \tag{3.4}$$

with the exponentially suppressed VEV.

Consider the limit $\sigma \ll 1$ for a fixed value r. In accordance with eq. (2.6) this corresponds to large values of the scalar curvature and, hence, to strong gravitational fields. To satisfy condition (2.13) we will assume that $\xi \geqslant 0$. For $\xi > 0$, from eq. (2.12) one has $\nu_l \gg 1$, and after introducing in eq. (3.2) a new integration variable $y = z/\nu_l$, we can replace the modified Bessel function by their uniform asymptotic expansions for large values of the order. The main contribution to the sum over l comes from the summand with $l = 0$, and the core induced VEV $\langle\varphi^2\rangle_c$ is suppressed by the factor $\exp\left[-(2/\sigma)\sqrt{n(n+1)\xi}\ln(r/a)\right]$. For $\xi = 0$ and $\sigma \ll 1$ for the terms with $l \neq 0$ one has $\nu_l \gg 1$ and the corresponding contribution is again exponentially small. For the summand with $l = 0$ to the leading order over σ we have $\nu_l = n/2$ and $\langle\varphi^2\rangle_c \sim 1/\sigma^{D-1}$. Hence, we conclude that in the limit of strong gravitational fields the behavior of the VEV $\langle\varphi^2\rangle_c$ is completely different for minimally and non-minimally coupled scalars.

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Now we turn to the investigation of the VEV of the energy-momentum tensor in the region $r > a$. Having the Wightman function and the VEV for the field square, these VEVs are evaluated on the base of the formula

$$\langle 0|T_{ik}|0\rangle = \lim_{x'\to x} \partial_i \partial'_k \langle 0|\varphi(x)\varphi(x')|0\rangle + \left[\left(\xi - \frac{1}{4}\right) g_{ik}\nabla_l\nabla^l - \xi\nabla_i\nabla_k - \xi R_{ik}\right] \langle 0|\varphi^2(x)|0\rangle. \tag{3.5}$$

Similar to the Wightman function, the components of the vacuum energy-momentum tensor can be presented in the decomposed form

$$\langle 0|T_{ik}|0\rangle = \langle 0_{\rm m}|T_{ik}|0_{\rm m}\rangle + \langle T_{ik}\rangle_c, \tag{3.6}$$

where $\langle 0_{\rm m}|T_{ik}|0_{\rm m}\rangle$ is the vacuum energy-momentum tensor for the geometry of a point-like global monopole and the part $\langle T_{ik}\rangle_c$ is induced by the core. In accordance with the problem symmetry both these tensors are diagonal. For massless fields the VEV of the energy-momentum tensor for the point-like global monopole geometry is investigated in refs. [5]–[8]. The corresponding renormalized components have the structure similar to that given for the field square:

$$\langle T_{ik}\rangle_{\rm m,ren} = \frac{1}{r^{D+1}}\left[q_{ik}^{(1)} + q_{ik}^{(2)} \ln(\mu r)\right], \tag{3.7}$$

where the coefficients $q_{ik}^{(1)}$, $q_{ik}^{(2)}$ depend only on the parameters σ and ξ, and $q_{ik}^{(2)} = 0$ for D being an even number. Substituting the expressions of the Wightman function and the VEV of the field square into formula (3.5), for the part of the energy-momentum tensor induced by the non-trivial core structure one obtains (no summation over i)

$$\langle T_i^k\rangle_c = -\frac{\sigma^{1-D}\delta_i^k}{2\pi r^n S_D}\sum_{l=0}^{\infty} D_l \int_m^\infty dz\, z^3 \frac{\tilde{I}_{\nu_l}(za)}{\tilde{K}_{\nu_l}(za)} \frac{F_{\nu_l}^{(i)}\left[K_{\nu_l}(zr)\right]}{\sqrt{z^2-m^2}}, \quad r > a, \tag{3.8}$$

where for a given function $f(y)$ the notations

$$F_{\nu_l}^{(0)}\left[f(y)\right] = (1-4\xi)\left[f'^2(y) - \frac{n}{y}f(y)f'(y) + \left(\frac{\nu_l^2}{y^2} - \frac{1+4\xi-2(mr/y)^2}{1-4\xi}\right)f^2(y)\right], \tag{3.9}$$

$$F_{\nu_l}^{(1)}\left[f(y)\right] = f'^2(y) + \frac{\tilde{\xi}}{y}f(y)f'(y) - \left(1 + \frac{\nu_l^2 + \tilde{\xi}n/2}{y^2}\right)f^2(y), \tag{3.10}$$

$$F_{\nu_l}^{(i)}\left[f(y)\right] = (4\xi-1)f'^2(y) - \frac{\tilde{\xi}}{y}f(y)f'(y) + \left[4\xi - 1 + \frac{\nu_l^2(1+\tilde{\xi}) + \tilde{\xi}n/2}{(n+1)y^2}\right]f^2(y), \tag{3.11}$$

are introduced with $\tilde{\xi} = 4(n+1)\xi - n$ and in eq. (3.11) $i = 2, 3, \ldots, D$. It can be seen that components (3.8) satisfy the continuity equation $\nabla_k\langle T_i^k\rangle_c = 0$, which for the geometry under consideration takes the form

$$r\frac{d}{dr}\langle T_1^1\rangle_c + (D-1)\left(\langle T_1^1\rangle_c - \langle T_2^2\rangle_c\right) = 0. \tag{3.12}$$

The core induced part $\langle T_i^k\rangle_c$ are finite everywhere outside the core, $r > a$, and diverge on the core boundary. Near this boundary the main contribution comes from large values

l and to find the corresponding asymptotic behavior we can use the uniform asymptotic expansions for the modified Bessel functions. To the leading order one finds $\langle T_i^k \rangle_c \sim 1/(r-a)^{\beta_2}$ for the energy density and the azimuthal stress and

$$\langle T_1^1 \rangle_c \approx -\frac{D-1}{\beta_2 - 1}(r/a-1)\langle T_2^2 \rangle_c, \tag{3.13}$$

with $\beta_2 \leqslant D+1$. An exception is the special case of the core model for which the leading term in the uniform asymptotic expansion for the function $\tilde{K}_{\nu_l}(za)$ vanishes. For large distances from the core boundary, $r \gg a$, and for a massless scalar field the main contribution into the VEV $\langle T_i^k \rangle_c$ comes from the $l=0$ summand. Under the assumptions $\nu_0 \neq 0$ and $\mathcal{R}_0(a,0) \neq \pm\nu_0$, the leading term of the corresponding asymptotic expansion behaves like $\langle T_i^k \rangle_c \sim (a/r)^{2\nu_0+D+1}$. For a massive scalar field under the condition $mr \gg 1$, the main contribution into the integral over z in eq. (3.8) comes from the lower limit and by using the asymptotic formulae for the function $K_{\nu_l}(zr)$ for large values of the argument, to the leading order one finds

$$\langle T_0^0 \rangle_c \approx -\langle T_2^2 \rangle_c \approx (4\xi - 1)\frac{\sqrt{\pi} m^{3/2} e^{-2mr}}{4r^{D-1/2} S_D \sigma^{D-1}} \sum_{l=0}^{\infty} D_l \frac{\tilde{I}_{\nu_l}(ma)}{\tilde{K}_{\nu_l}(ma)}, \tag{3.14}$$

and the radial stress is suppressed by an additional factor $1/mr$.

Now let us consider the VEV of the energy-momentum tensor in the limit $\sigma \ll 1$ for a fixed $r > a$. For $\xi > 0$ by the calculations similar to those given above for the field square, one finds that the core induced VEV are suppressed by the factor

$$\exp\left[-(2/\sigma)\sqrt{n(n+1)\xi}\,\ln(r/a)\right]$$

and the vacuum stresses are strongly anisotropic: $\langle T_1^1 \rangle_c / \langle T_2^2 \rangle_c \sim \sigma$. For a minimally coupled scalar, $\xi = 0$, the leading term of the asymptotic expansion over σ comes from the $l=0$ summand in eq. (3.8) with $\nu_l = n/2$. This term behaves as σ^{1-D}.

4. Core with an infinitely thin shell

The results considered in the previous section can be generalized to the models where an additional infinitely thin spherical shell located at $r=a$ is present with the surface energy-momentum tensor τ_i^k. We denote by n^i the normal to the shell normalized by the condition $n_i n^i = -1$, assuming that it points into the bulk on both sides. From the Israel matching conditions one has

$$\{K_{ik} - K h_{ik}\} = 8\pi G \tau_{ik}, \tag{4.1}$$

where the curly brackets denote summation over each side of the shell, $h_{ik} = g_{ik} + n_i n_k$ is the induced metric on the shell, $K_{ik} = h_i^r h_k^s \nabla_r n_s$ its extrinsic curvature and $K = K_i^i$. For the region $r \leqslant a$ one has $n_i = \delta_i^1 e^{v(r)}$ and the non-zero components of the extrinsic curvature are given by the formulae

$$K_0^0 = -u'(r)e^{-v(r)},\ K_i^j = -\delta_i^j w'(r) e^{-v(r)},\ i = 2, 3, \ldots,\ r = a-. \tag{4.2}$$

The corresponding expressions for the region $r \geqslant a$ are obtained by taking $u(r) = v(r) = 0$, $w(r) = \ln(\sigma r)$ and changing the signs for the components of the extrinsic curvature tensor. Now from the matching conditions (4.1) we find (no summation over i)

$$u'(a-) = 8\pi G \left[\tau_i^i - \frac{D-2}{D-1}\tau_0^0 \right], \; i = 2, 3, \ldots, \tag{4.3}$$

$$w'(a-) = \frac{1}{a} + \frac{8\pi G}{D-1}\tau_0^0 \tag{4.4}$$

where $f'(a-)$ is understood in the sense $\lim_{r \to a-0} f'(r)$. The discontinuity of the functions $u'(r)$ and $w'(r)$ at $r = a$ leads to the delta function term

$$2 \left[u'(a-) + (D-1) \left(w'(a-) - 1/a \right) \right] \delta(r-a) \tag{4.5}$$

in the Ricci scalar and, hence, in the equation (2.11) for the radial eigenfunctions. Note that the expression in the square brackets is related to the surface energy-momentum tensor by the formula

$$u'(a-) + (D-1) \left(w'(a-) - 1/a \right) = \frac{8\pi G}{D-1}\tau, \tag{4.6}$$

where τ is the trace of the surface energy-momentum tensor.

Due to the delta function term in the equation for the radial eigenfunctions, these functions have a discontinuity in their slope at $r = a$. The corresponding jump condition is obtained by integrating the equation (2.11) through the point $r = a$:

$$f_l'(a+) - f_l'(a-) = \frac{16\pi G\xi}{D-1}\tau f_l(a). \tag{4.7}$$

Now the coefficients in the formulae (2.14) for the eigenfunctions are determined by the continuity condition for the radial eigenfunctions and by the jump condition for their radial derivative. It can be seen that the corresponding eigenfunctions are given by the same formulae (2.17) and (2.22) with the new barred notation

$$\bar{F}(z) \equiv zF'(z) - \left[\frac{n}{2} + \frac{16\pi G\xi}{D-1}a\tau + a\frac{R_l'(a,\lambda)}{R_l(a,\lambda)} \right] F(z). \tag{4.8}$$

Consequently the parts in the Wightman function, in the VEVs of the field square and the energy-momentum tensor induced by the core of the finite thickness, are given by formula (2.29), (3.2) and (3.8), where the tilted notation is defined by eq. (2.30) with the function

$$\mathcal{R}_l(a,z) = \frac{n}{2} + \frac{16\pi G\xi}{D-1}a\tau + a\frac{R_l'(a, ze^{\pi i/2}/a)}{R_l(a, ze^{\pi i/2}/a)}. \tag{4.9}$$

The trace of the surface energy-momentum tensor in this expression is related to the components of the metric tensor inside the core by formula (4.6).

5. Flower-pot model for global monopole

As an application of the general results given above let us consider a simple example of the core model assuming that the spacetime inside the core is flat. The corresponding model

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for the cosmic string core was considered in refs. [21 − 23] and following these papers we will refer to this model as flower-pot model. Taking $u(r) = v(r) = 0$ from the zero curvature condition one finds $e^{w(r)} = r + \text{const}$. The value of the constant here is found from the continuity condition for the function $w(r)$ at the boundary which gives const $= (\sigma - 1)a$. Hence, the interior line element has the form

$$ds^2 = dt^2 - dr^2 - [r + (\sigma - 1)a]^2 \, d\Omega_D^2. \tag{5.1}$$

In terms of the radial coordinate r the origin is located at $r = (1-\sigma)a$. From the matching conditions (4.3), (4.4) we find the corresponding surface energy-momentum tensor with the non-zero components

$$\tau_0^0 = \left(\frac{1}{\sigma} - 1\right) \frac{D-1}{8\pi Ga}, \; \tau_i^k = \frac{D-2}{D-1} \tau_0^0 \delta_i^k, \; i = 2, 3, \ldots. \tag{5.2}$$

The corresponding surface energy density is positive for the global monopole with $\sigma < 1$. After this brief review, let us analyze for this model the influence of the monopole's core on the vacuum polarization effects. We will consider the exterior and interior regions separately.

5.1 Exterior region

In the region inside the core the radial eigenfunctions regular at the origin are the functions:

$$R_l(r, \lambda) = C_l \frac{J_{l+n/2}(\lambda \tilde{r})}{\tilde{r}^{n/2}}, \tag{5.3}$$

where $\tilde{r} = r + (\sigma - 1)a$ is the standard Minkowskian radial coordinate, $0 \leqslant \tilde{r} \leqslant \sigma a$. In appendix we show that in the flower-pot model no bound states exist. Note that for an interior Minkowskian observer the radius of the core is σa. The normalization coefficient C_l is found from the condition (2.22):

$$C_l^2 = \frac{2\lambda J_{l+n/2}^{-2}(\lambda\sigma a)}{\pi^2 \sigma \omega N(m_k) \left[\bar{J}_{\nu_l}^2(\lambda a) + \bar{Y}_{\nu_l}^2(\lambda a)\right]}, \tag{5.4}$$

with the barred notation for the cylindrical functions

$$\bar{F}(z) \equiv zF'(z) - \left[\alpha_\sigma + z\frac{J'_{l+n/2}(z\sigma)}{J_{l+n/2}(z\sigma)}\right] F(z) \tag{5.5}$$

and

$$\alpha_\sigma = \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)[n - 4\xi(n+1)]\,. \tag{5.6}$$

Note that $\bar{J}_{\nu_l}(\lambda a) = 0$ for $\sigma = 1$. Hence, the parts in the Wightman function, in the VEVs of the field square and the energy-momentum tensor due the non-trivial structure of the core in the flower-pot model, are given by formulae (2.29), (3.2) and (3.8) respectively, where the tilted notations for the modified Bessel functions are defined by (2.30) with the coefficient

$$\mathcal{R}_l(a, z) = \alpha_\sigma + z\frac{I'_{l+n/2}(z\sigma)}{I_{l+n/2}(z\sigma)}. \tag{5.7}$$

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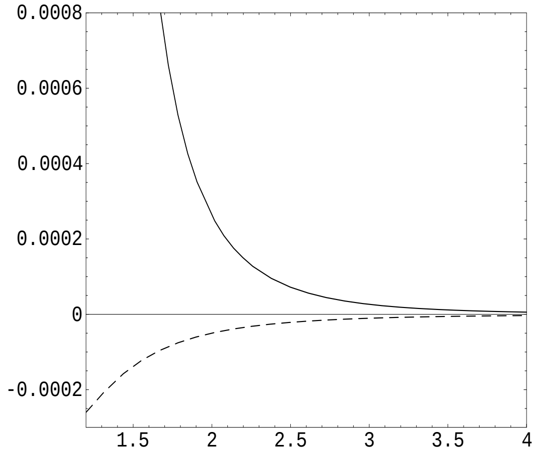

Figure 1: The expectation value $a^{D-1}\langle\varphi^2\rangle_c$ induced by the non-trivial core structure in the region outside the core for $D = 3$ massless scalar field as a function of r/a in flower-pot model with $\sigma = 0.5$. The full/dashed curves correspond to minimally/conformally coupled scalars.

For $\sigma = 1$ one has $\tilde{I}_{\nu_l}(z) = 0$ and as we could expect the VEVs vanish. Using the value for the standard integral involving the product of the functions K_ν given in ref. [26], in the case of a massless scalar field the leading term for the asymptotic expansion over a/r can be presented in the form

$$\langle\varphi^2\rangle_c \approx -\frac{\nu_0\Gamma(2\nu_0 + 1/2)\Gamma(\nu_0 + 1/2)\mathcal{A}_n}{2^{2\nu_0+1}(a\sigma)^{D-1}S_D\Gamma^3(\nu_0 + 1)}\left(\frac{a}{r}\right)^{2\nu_0+D-1}, \tag{5.8}$$

where

$$\mathcal{A}_n = \frac{n\sigma - 4\xi(n + 1)(\sigma - 1) - 2\sigma\nu_0}{n\sigma - 4\xi(n + 1)(\sigma - 1) + 2\sigma\nu_0}. \tag{5.9}$$

Note that for a minimally coupled scalar $\mathcal{A}_n = 0$ and the presented leading term vanishes. In figure 1 we have plotted the dependence of the part in the VEV of the field square induced by the core as a function on the rescaled radial coordinate for minimally and conformally coupled $D = 3$ massless scalar fields in the flower-pot model with $\sigma = 0.5$

Now let us analyze the VEV of the energy-momentum tensor given by eq. (3.8) with the tilted notation given by (2.30), (5.7). For large distances from the core, $r \gg a$, the main contribution into the VEV of the energy-momentum tensor for a massless scalar field comes from the $l = 0$ summand. Under the assumption $\nu_0 \neq 0$ the leading terms of the asymptotic expansions have the form (no summation over i)

$$\langle T_i^k\rangle_c \approx -\frac{2^{-2\nu_0}\sigma^{1-D}\mathcal{A}_n\delta_i^k}{\pi\nu_0 S_D\Gamma^2(\nu_0)a^{D+1}}\left(\frac{a}{r}\right)^{2\nu_0+D+1}\int_0^\infty dz z^{2\nu_0+2}F_{\nu_0}^{(i)}[K_{\nu_0}(z)]. \tag{5.10}$$

The integrals in this formula can be evaluated using the value for the integrals involving the product of the functions K_ν given in ref. [26]. As we see, for $\nu_0 > 0$ and for large distances from the sphere the vacuum energy-momentum tensor is dominated by the part corresponding to the point-like monopole.

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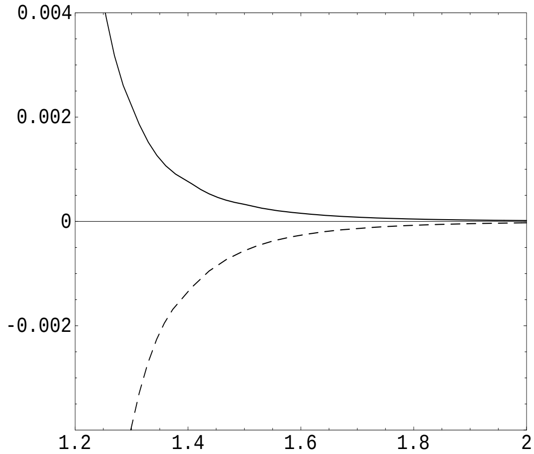

Figure 2: The expectation value of the energy density, $a^{D+1}\langle T_0^0\rangle_c$ induced in the region outside the core for $D=3$ massless scalar field as a function of r/a in flower-pot model with $\sigma=0.5$. The full/dashed curves correspond to minimally/conformally coupled scalars.

As it has been mentioned above on the core surface the VEVs diverge. For the region near the core the main contribution comes from large values of l. By using the uniform asymptotic expansions for the modified Bessel functions it can be seen that to the leading order $\langle\varphi^2\rangle_c \sim (r-a)^{2-D}$ and the components of the vacuum energy-momentum tensor behave as $(r-a)^{-D}$ for the energy density and the azimuthal stress and as $(r-a)^{1-D}$ for the radial stress. Due to surface divergencies near the surface the total vacuum energy-momentum tensor is dominated by the parts induced by the finite thickness of the core. As an illustration, in figure 2 we have presented the dependence of the core-induced vacuum energy density as a function on the radial coordinate for $D=3$ minimally and conformally coupled massless scalar fields in the flower-pot model with $\sigma=0.5$.

5.2 Interior region

Now let us consider the vacuum polarization effects inside the core for the flower-pot model. The corresponding eigenfunctions have the form given by eq. (2.9) with $f_l(r)=R_l(r,\lambda)$ and the function $R_l(r,\lambda)$ is defined by formula (5.3). Substituting the eigenfunctions into the mode sum formula for the corresponding Wightman function one finds

$$\begin{aligned}\langle 0|\varphi(x)\varphi(x')|0\rangle &= \frac{2}{\pi^2 n\sigma S_D}\sum_{l=0}^{\infty}\frac{2l+n}{(\tilde{r}\tilde{r}')^{n/2}}C_l^{n/2}(\cos\theta)\\ &\times\int_0^\infty d\lambda\,\frac{\lambda J_{l+n/2}^{-2}(\lambda\sigma a)}{\sqrt{\lambda^2+m^2}}\frac{J_{l+n/2}(\lambda\tilde{r})J_{l+n/2}(\lambda\tilde{r}')}{\bar{J}_{\nu_l}^2(\lambda a)+\bar{Y}_{\nu_l}^2(\lambda a)}e^{i\sqrt{\lambda^2+m^2}(t'-t)}. \quad (5.11)\end{aligned}$$

To find the renormalized VEVs of the field square and the energy-momentum tensor we need to evaluate the difference between this function and the corresponding function for the Minkowski bulk:

$$\langle\varphi(x)\varphi(x')\rangle_{\rm sub} = \langle 0|\varphi(x)\varphi(x')|0\rangle - \langle 0_M|\varphi(x)\varphi(x')|0_M\rangle. \quad (5.12)$$

The appropriate form for the Minkowskian part is obtained from eq. (2.25) taking $\sigma = 1$ and replacing $r \to \tilde{r}$. By using the corresponding formula, for the subtracted Wightman function one finds

$$\langle \varphi(x)\varphi(x') \rangle_{\rm sub} = \frac{2}{\pi^2 n S_D} \sum_{l=0}^{\infty} \frac{2l+n}{(\tilde{r}\tilde{r}')^{n/2}} C_l^{n/2}(\cos\theta) \int_0^{\infty} d\lambda \, \frac{\lambda e^{i\sqrt{\lambda^2+m^2}(t'-t)}}{\sqrt{\lambda^2+m^2}}$$
$$\times J_{l+n/2}(\lambda\tilde{r}) J_{l+n/2}(\lambda\tilde{r}') \left[\frac{J_{l+n/2}^{-2}(\lambda\sigma a)/\sigma}{\bar{J}_{\nu_l}^2(\lambda a) + \bar{Y}_{\nu_l}^2(\lambda a)} - \frac{\pi^2}{4} \right], \quad (5.13)$$

where the barred notation is defined by eq. (5.5). The integral in this formula is slowly convergent and the integrand is highly oscillatory. In order to transform the expression for the subtracted Wightman function into more convenient form, we note that the following identity takes place

$$\frac{1}{2} \sum_{s=1,2} \frac{C\{H_{l+n/2}^{(s)}(\sigma z), J_{\nu_l}(z)\}}{J_{l+n/2}(\sigma z)\bar{J}_{\nu_l}(z)} = 1, \quad (5.14)$$

where we have introduced the notation

$$C\left\{f(\sigma z), g(z)\right\} = z f(\sigma z) g'(z) - \left[\alpha_\sigma f(\sigma z) + z f'(\sigma z)\right] g(z). \quad (5.15)$$

Note that in terms of this notation one has

$$J_{l+n/2}(\sigma z)\bar{F}(z) = C\{J_{l+n/2}(\sigma z), F(z)\}. \quad (5.16)$$

We add the left-hand side of eq. (5.14) with $z = \lambda a$ as a coefficient to the term $\pi^2/4$ in the square brackets of eq. (5.13). After this replacement the term in the square brackets is written in the form

$$\frac{J_{l+n/2}^{-2}(\sigma z)/\sigma}{\bar{J}_{\nu_l}^2(z) + \bar{Y}_{\nu_l}^2(z)} - \frac{\pi^2}{4} = \sum_{s=1,2} \frac{1}{2C\{J_{l+n/2}(\sigma z), J_{\nu_l}(z)\}}$$
$$\times \left[\frac{1/\sigma}{C\{J_{l+n/2}(\sigma z), H_{\nu_l}^{(s)}(z)\}} - \frac{\pi^2}{4} C\{H_{l+n/2}^{(s)}(\sigma z), J_{\nu_l}(z)\} \right]. \quad (5.17)$$

Note that both terms in the sum over s on the right of this relation are separately regular at the zeros of the function $C\{J_{l+n/2}(\sigma\lambda a), J_{\nu_l}(\lambda a)\}$. Substituting (5.17) into formula (5.13) we rotate the integration contour in the complex plane λ by the angle $\pi/2$ for $s = 1$ and by the angle $-\pi/2$ for $s = 2$. Under the condition $\tilde{r} + \tilde{r}' + |t - t'| < 2\sigma a$ the contribution from the semicircle with the radius tending to infinity vanishes. Note that as we consider the points inside the core this condition is satisfied in the coincidence limit. The integrals over the segments $(0, im)$ and $(0, -im)$ of the imaginary axis cancel out and after introducing the modified Bessel functions the subtracted Wightman function can be presented in the form

$$\langle \varphi(x)\varphi(x') \rangle_{\rm sub} = -\frac{1}{\pi n S_D} \sum_{l=0}^{\infty} \frac{2l+n}{(\tilde{r}\tilde{r}')^{n/2}} C_l^{n/2}(\cos\theta) \int_m^{\infty} dz \, \frac{z U_l(\sigma, za)}{\sqrt{z^2-m^2}}$$

$$\times I_{l+n/2}(z\tilde{r})I_{l+n/2}(z\tilde{r}')\cosh\left[\sqrt{z^2-m^2}(t'-t)\right], \tag{5.18}$$

with the notation

$$U_l(\sigma,z) = \frac{1/\sigma + C\{I_{l+n/2}(\sigma z), K_{\nu_l}(z)\}C\{K_{l+n/2}(\sigma z), I_{\nu_l}(z)\}}{C\{I_{l+n/2}(\sigma z), I_{\nu_l}(z)\}C\{I_{l+n/2}(\sigma z), K_{\nu_l}(z)\}}. \tag{5.19}$$

For points away from the core boundary the integral is exponentially convergent in the coincidence limit and this formula is convenient for the calculation of the VEVs of the field square and the energy-momentum tensor.

Having the subtracted Wightman function we can evaluate the renormalized VEV of the field square by taking the coincidence limit of the arguments in eq. (5.18):

$$\langle\varphi^2\rangle_{\rm ren} = -\frac{1}{\pi S_D \tilde{r}^n}\sum_{l=0}^{\infty} D_l \int_m^{\infty} dz \frac{zU_l(\sigma,za)}{\sqrt{z^2-m^2}}\, I^2_{l+n/2}(z\tilde{r}). \tag{5.20}$$

So, by this result we can see that although the spacetime inside the core is Minkowski one, there exists a vacuum polarization induced by the non-trivial topology of the spacetime in the exterior region. In the limit $r \to 0$ the main contribution into eq. (5.20) comes from the $l = 0$ summand with the leading term

$$\langle\varphi^2\rangle_{\rm ren} \approx -\frac{2^{1-D}}{\pi^{D/2+1}\Gamma(D/2)}\int_m^{\infty} dz\, z^{D-1}\frac{U_0(\sigma,za)}{\sqrt{z^2-m^2}}, \tag{5.21}$$

and at the monopole center the renormalized VEV of the field square acquires a non-vanishing regular contribution. For the points on the core surface the VEV given by eq. (5.20) diverges like $1/(a-r)^{D-2}$. Now let us consider the limiting case $\sigma \ll 1$ under the fixed value σa which is the core radius for an internal Minkowskian observer. Introducing in eq. (5.20) a new integration variable $y = \sigma a z$ and by making use of the uniform asymptotic expansions for the modified Bessel functions with the index ν_l, we can see that to the leading order

$$\langle\varphi^2\rangle_{\rm ren} \approx -\frac{1}{\pi S_D \tilde{r}^n}\sum_{l=0}^{\infty} D_l \int_m^{\infty} dz \frac{zU_{l+n/2}(z\sigma a)}{\sqrt{z^2-m^2}}\,\frac{I^2_{l+n/2}(z\tilde{r})}{I^2_{l+n/2}(z\sigma a)}, \tag{5.22}$$

where we have introduced the notation

$$U_\nu(y) = \frac{2\sqrt{\eta_1+y^2}}{\eta_1+y^2-\left[\eta_2+yI'_\nu(y)/I_\nu(y)\right]^2} - I_\nu(y)K_\nu(y), \tag{5.23}$$

with

$$\eta_1 = l\,(l+n) + n(n+1)\xi,\ \eta_2 = 2\xi(n+1) - n/2. \tag{5.24}$$

Hence, in the limit $\sigma \to 0$ for a fixed radius of the core, σa, the part in the renormalized VEV of the field square inside the core tends to the finite limiting value. For large values of the mass, assuming that $m(\sigma a - \tilde{r}) \gg 1$, it can be seen that $\langle\varphi^2\rangle_{\rm ren}$ is suppressed by the factor $e^{-2m(\sigma a-\tilde{r})}$. In figure 3 we have plotted the renormalized VEV $\langle\varphi^2\rangle_{\rm ren}$ inside the core of the flower-pot model with $\sigma = 0.5$ as a function of $\tilde{r}/\sigma a$ for minimally and conformally

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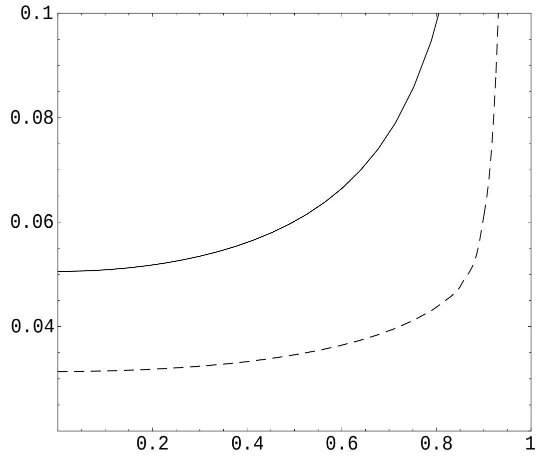

Figure 3: The expectation value $a^{D-1}\langle\varphi^2\rangle_{\rm ren}$ inside the core for $D=3$ massless scalar field as a function of $\tilde{r}/\sigma a$ in the flower-pot model with $\sigma=0.5$. The full/dashed curves correspond to minimally/conformally coupled scalars.

coupled massless scalars. Again we can observe that there exists a strong dependence of this quantity on the curvature coupling parameter.

The renormalized VEV of the energy-momentum tensor is found by using the formula (3.5) with the subtracted Wightman functions. This leads to the following formula (no summation over i)

$$\langle T_i^k\rangle_{\rm ren} = -\frac{\delta_i^k}{2\pi S_D \tilde{r}^n}\sum_{l=0}^{\infty} D_l \int_m^{\infty} dz \frac{z^3 U_l(\sigma, za)}{\sqrt{z^2-m^2}} F^{(i)}_{l+n/2}[I_{l+n/2}(z\tilde{r})], \tag{5.25}$$

where the functions $F^{(i)}_{l+n/2}[f(y)]$ are defined by relations (3.9)-(3.11) with the replacement $\nu_l \to l+n/2$. At the core center the nonzero contribution to VEV (5.25) comes from the summands with $l=0$ and $l=1$ and one has

$$\begin{aligned}\langle T_0^0\rangle_{\rm ren} &= \frac{1}{2^D\pi^{D/2+1}\Gamma(D/2)}\int_m^{\infty}\frac{z^{D+1}dz}{\sqrt{z^2-m^2}} \\ &\times\left[\left(4\xi+1-2\frac{m^2}{z^2}\right)U_0(\sigma,za)+(4\xi-1)U_1(\sigma,za)\right], \end{aligned}\tag{5.26}$$

$$\begin{aligned}\langle T_1^1\rangle_{\rm ren} &= \langle T_2^2\rangle_{\rm ren} = \frac{1}{2^D\pi^{D/2+1}D\Gamma(D/2)}\int_m^{\infty}\frac{z^{D+1}dz}{\sqrt{z^2-m^2}} \\ &\times\left[\left(\tilde{\xi}-2\right)U_0(\sigma,za)+\tilde{\xi}U_1(\sigma,za)\right], \end{aligned}\tag{5.27}$$

Note that for the conformally coupled massless scalar at the center one has $\langle T_0^0\rangle_{\rm ren} = -D\langle T_1^1\rangle_{\rm ren}$. This can also be obtained directly from the zero trace condition. Near the core surface the components of the vacuum energy-momentum tensor behave as $(a-r)^{-D}$ for the energy density and the azimuthal stress and as $(a-r)^{1-D}$ for the radial stress. As in the case of the field square, in the limit $\sigma\to 0$ for a fixed radius of the core radius σa, the part in the vacuum energy-momentum tensor induced by the non-trivial core tends to

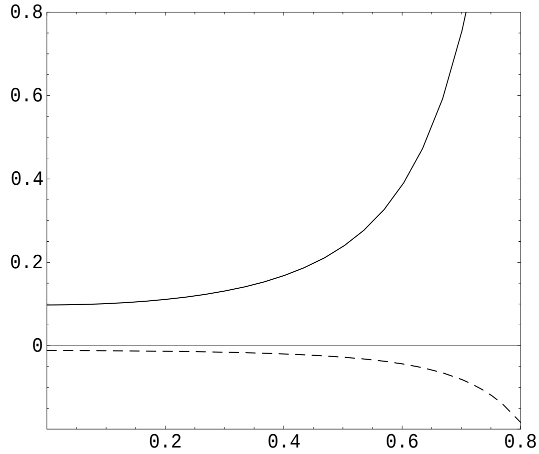

Figure 4: The renormalized energy density, $a^{D+1}\langle T_0^0\rangle_{\rm ren}$ inside the core for $D=3$ massless scalar field as a function of $\tilde{r}/\sigma a$ in the flower-pot model with $\sigma=0.5$. The full/dashed curves correspond to minimally/conformally coupled scalars.

the finite limiting value. This limiting value is obtained from formula (5.25) by making the replacement $U_l(\sigma, za) \to U_{l+n/2}(z\sigma a)/I^2_{l+n/2}(z\sigma a)$. As in the case of the field square, for large values of the mass for the field quanta the VEV $\langle T_i^k\rangle_{\rm ren}$ is exponentially suppressed by the factor $e^{-2m(\sigma a-\tilde{r})}$. The dependence of the renormalized interior vacuum energy density on the radial coordinate is presented in figure 4 for minimally and conformally coupled massless scalar field in $D=3$ for the geometry of a global monopole with $\sigma=0.5$.

6. Conclusion

In the present paper we have considered the one-loop vacuum effects for a massive scalar field with general curvature coupling parameter on background of the $(D+1)$-dimensional global monopole with non-trivial core structure. The previous papers on the investigation of the vacuum polarization by the gravitational field of the global monopole are concerned with the idealized point-like model, where the curvature has singularity at the origin. The exception is the ref. [18], where the vacuum densities for a massless scalar field are studied outside the monopole core with the interior de Sitter geometry. Here we consider the general spherically symmetric static model of the core with finite thickness, described by the line element (2.2), and investigate the vacuum properties in both exterior and interior regions. Among the most important characteristics of these properties, which carry an information about the core structure, are the VEVs for the field square and the energy-momentum tensor. In order to obtain these expectation values we first construct the positive frequency Wightman function. In the region outside the core this function is presented as a sum of two distinct contributions. The first one corresponds to the Wightman function for the geometry of a point-like global monopole and the second one is induced by the non-trivial structure of the monopole's core. The latter is given by formula (2.29), where the tilted notation is defined by formula (2.30) with the coefficient from (2.31) for the model without an infinitely thin spherical shell on the boundary of the core. This coefficient is determined by the radial part of the interior eigenfunctions and describes the influence of the core properties on the vacuum characteristics in the exterior region. In the case of the core model with a thin shell on the boundary the derivatives of the metric tensor components are discontinuous on the core surface. This leads to the delta function type contribution to the Ricci scalar and, hence to the equation for the radial eigenfunctions in the case of the non-minimally coupled scalar field. As a result, the radial eigenfunctions have a discontinuity in their slope at the core boundary. This leads to an additional term in the coefficient of the tilted notation which is proportional to the trace of the surface energy-momentum tensor (see eq.(4.9)).

By using the formula for the Wightman function, in section 3 we have investigated the influence of the non-trivial core structure on the VEVs of the field square and the energy-momentum tensor. As in the exterior region the local geometry is the same as that in the point-like global monopole model, the presence of the core does not lead to additional divergences for the points outside the core. As a result, the parts in these VEVs induced by the core are directly obtained from the corresponding part of the Wightman function for the case of the field square and by applying on this function a certain second-order differential operator and taking the coincidence limit for the energy-momentum tensor. These parts are given by formulae (3.2) and (3.8) for the field square and the energy-momentum tensor respectively. They diverge as the boundary of the core is approached. The surface divergences in the VEVs of the local observables are well-known in quantum field theory with boundaries and are investigated for various boundary geometries. We have investigated the asymptotic behavior of the core induced VEVs near the core boundary and at large distances from the core. In particular, at large distances and for a massless scalar field with $\nu_0 > 0$, the ratio of the core induced and the point-like monopole parts decay as $(r/a)^{2\nu_0}$ for the both field square and the energy-momentum tensor. For the special case with $\nu_0 = 0$ this ratio decays logarithmically and long-range effects of the monopole's core appear. In the limit of strong gravitational fields corresponding to small values of the parameter σ, the behavior of the core induced parts is completely different for minimally and non-minimally coupled fields. The corresponding VEVs are suppressed by the factor $\exp[-(2/\sigma)\sqrt{n(n+1)\xi}\ln(a/r)]$ for the non-minimally coupled scalar and behave like σ^{1-D} for the minimally coupled field.

As an example of the application of the general results, in section 5 we have considered a simple core model with a flat spacetime inside the core, so called flower-pot model. The corresponding surface energy-momentum tensor on the boundary of the core is obtained from the matching conditions and has the form given by eq. (5.2). The core induced parts of the exterior VEVs in this model are obtained from the general results by taking the function in the coefficient of the tilted notation form eq. (5.7). For the flower-pot model we have also investigated the vacuum densities inside the core. Though the spacetime geometry inside the core is Monkowskian, the non-trivial topology of the exterior region induces vacuum polarization effects in this region as well. In order to find the corresponding renormalized VEVs of the field square and the energy-momentum tensor we have derived a closed formula, eq. (5.18), for the difference of the interior Wightman function and the

Wightman function for the Minkowski spacetime. The subtracted function is finite in the coincidence limit and can be directly used for the evaluation of the VEVs of the field square and the energy-momentum tensor. The latter quantities are given by formulae (5.20) and (5.25). As in the case of the exterior region, we have considered various limiting cases when the general formulae are simplified. In particular, we have shown that in the limit $\sigma \ll 1$ under the fixed value σa, which is the core radius for an internal Minkowskian observer, the renormalized VEVs of the field square and the energy-momentum tensor tend to finite limiting values.

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A. Contribution of bound states

In this appendix we consider the contribution of possible bound states into the VEVs. For this states the quantity λ is purely imaginary, $\lambda = i\eta$, and the corresponding radial eigenfunction in the region $r > a$ is the function $A_{bl} r^{-n/2} K_{\nu_l}(\eta r)$ with the normalization coefficient A_{bl}. To have a stable ground state we will assume that $\eta < m$. From the continuity of the eigenfunctions at $r = a$ one has

$$R_l(a, i\eta) = A_{bl} a^{-n/2} K_{\nu_l}(\eta a), \tag{A.1}$$

and from the continuity of the radial derivative we see that the possible bound states are solutions of the equation

$$\tilde{K}_{\nu_l}(\eta a) = 0, \tag{A.2}$$

with the notation from (2.30). The normalization condition for the bound states is as follows:

$$\int_{r_0}^{a} dr\, e^{-u+v+(D-1)w} R_l^2(r, i\eta) + A_{bl} \sigma^{D-1} \int_a^\infty dr\, r K_{\nu_l}^2(\eta r) = \frac{1}{2\omega N(m_k)}, \tag{A.3}$$

from which the normalization constant can be found. In order to evaluate the integrals in this formula we note that for the solution $f_{\omega l}(r)$ to radial equation (2.11) the following formula takes place

$$\int dr\, e^{-u+v+(D-1)w} f_{\omega l}(r) f_{\omega_1 l}(r) = \frac{e^{-u+v+(D-1)w}}{\omega_1^2 - \omega^2} \left[f'_{\omega l}(r) f_{\omega_1 l}(r) - f_{\omega l}(r) f'_{\omega_1 l}(r) \right]. \tag{A.4}$$

In particular, in the limit $\omega_1 \to \omega$ one finds

$$\int dr\, e^{-u+v+(D-1)w} f_{\omega l}^2(r) = \frac{e^{-u+v+(D-1)w}}{2\omega} \left[f'_{\omega l}(r) \frac{\partial}{\partial \omega} f_{\omega l}(r) - f_{\omega l}(r) \frac{\partial}{\partial \omega} f'_{\omega l}(r) \right]. \tag{A.5}$$

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Applying to the integrals in eq. (A.3) this formula and using the continuity of the radial eigenfunctions at $r = a$, for the normalization coefficient one finds

$$A_{bl}^2 = -\frac{\eta\sigma^{1-D}\tilde{I}_{\nu_l}(\eta a)}{\omega N(m_k)(\partial/\partial\eta)\tilde{K}_{\nu_l}(\eta a)}. \tag{A.6}$$

To obtain this formula we have used the relation

$$K_{\nu_l}(\eta a) = 1/\tilde{I}_{\nu_l}(\eta a), \tag{A.7}$$

valid for the solutions of eq. (A.2). As a result, for the contribution of the bound state with $\lambda = i\eta$ to the Wightman function we have the formula

$$\begin{aligned}\langle\varphi(x)\varphi(x')\rangle_{\rm bs} = & -\frac{\sigma^{1-D}}{nS_D}\sum_{l=0}^{\infty}\frac{2l+n}{(rr')^{n/2}}C_l^{n/2}(\cos\theta) \\ & \times\frac{\eta\tilde{I}_{\nu_l}(\eta a)}{(\partial/\partial\eta)\tilde{K}_{\nu_l}(\eta a)}\frac{e^{i(t'-t)\sqrt{m^2-\eta^2}}}{\sqrt{m^2-\eta^2}}K_{\nu_l}(\eta r)K_{\nu_l}(\eta r').\end{aligned} \tag{A.8}$$

In the case when several bound states are present the sum of their separate contributions should be taken. Now the Wightman function is the sum of the part coming from the modes with real λ given by eq. (2.28) and of the part coming from the bound states given by eq. (A.8). In order to transform the first part we again rotate the integration contour in eq. (2.28) by the angle $\pi/2$ for $s = 1$ and by the angle $-\pi/2$ for $s = 2$. But now we should take into account that the integrand has poles at $\lambda = \pm i\eta$ which are zeroes of the functions $\bar{H}_{\nu_l}^{(s)}(\lambda a)$ in accordance with eq. (A.2). Rotating the integration contour we will assume that the pole $(-1)^s i\eta$, $s = 1, 2$, on the imaginary axis is avoided by the semicircle $C_\rho^{(s)}$ in the right half plane with small radius ρ and with the center at this pole. The integration over these semicircles will give an additional contribution

$$-\frac{\sigma^{1-D}}{4nS_D}\sum_{l=0}^{\infty}\frac{2l+n}{(rr')^{n/2}}C_l^{n/2}(\cos\theta)\sum_{s=1}^{2}\int_{C_\rho^{(s)}}d\lambda\,\lambda\frac{e^{i\sqrt{\lambda^2+m^2}(t'-t)}}{\sqrt{\lambda^2+m^2}}\frac{\bar{J}_{\nu_l}(\lambda a)}{\bar{H}_{\nu_l}^{(s)}(\lambda a)}H_{\nu_l}^{(s)}(\lambda r)H_{\nu_l}^{(s)}(\lambda r'). \tag{A.9}$$

By evaluating the integrals in this formula it can be seen that this term exactly cancels the contribution (A.8) coming from the corresponding bound state (for the similar cancellation in the Casimir effect with Robin boundary condition see ref. [27]). Hence, we conclude that the formulae given above for the core induced parts in the VEVs are valid in the case of the presence of bound states as well.

In order to see the possibility for the appearance of bound states in the flower-pot model, we note that introducing a function $F_l(r) = r_*^{(D-1)/2} f_l(r)$ with $r_* = r$ outside the core and $r_* = \tilde{r}$ inside the core, equation (2.11) for the radial part of the eigenfunctions is written in the form of the Schrodinger equation. The corresponding effective potential is equal $[\nu_l^2 + (n-1)/4]/\sigma^2 r^2$ in the exterior region and $[l(l+n) + (n^2-1)/4]/\tilde{r}^2$ in the interior region. Under the conditions $\nu_l^2 \geqslant 0$ and $n > 0$ assumed earlier, the potential is non-negative and, hence, in the flower-pot model no bound states exist.

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